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FINANCE AND ENDOGENOUS GROWTH

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ABSTRACT

In a two-class growth model of Pasinetti (1962), there is no financial intermediary that mobilizes bank deposits to be lent to the capitalist class for physical investment. The absence of a capital market also precludes workers from buying capitalists' new issues of stocks and bonds to finance investment. Thus, the equilibrium rate of return to capital is independent of the saving rate of the working class—what Samuelson and Modigliani (1966) referred to as the Pasinetti paradox. In this paper's modified Pasinetti framework with endogenous growth, the equilibrium rate of return to capital is shown to be a function of all structural parameters, including both saving rates of the capitalist and working classes. Additionally, the modified model explains the recessionary dynamics of the 2007/2008 global and regional financial crises. Implications for growth policies are drawn.

Keywords: Pasinetti mode; Endogenous growth; Growth policies.
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I. INTRODUCTION

The Solow (1956) and Swan (1956), henceforth S-S, growth model has been, and still is, the workhorse of basic neoclassical growth theory. The S-S growth model has two distinguishing features: (a) it is a "real" model, i.e., it has no financial sector; and (b) all technical change is exogenous. I relax both features in a two-class neoclassical growth model with a highly simplified financial sector. The other innovation is to incorporate the learning-by-doing model in Villanueva (1994) to make Harrod-neutral technical change endogenous.[1]

Pasinetti (1962) developed a two-class model of income distribution involving the capitalist and working classes that led to what Samuelson and Modigliani (1966) referred to as the Pasinetti paradox, which says that the equilibrium rate of return to capital (capital's marginal product) is given by the ratio $\frac{n}{s_c}$, where n is the growth rate of labor adjusted for Harrod-neutral technical change and s_c is the saving rate of the capitalist class. Thus, the equilibrium rate of return to capital is independent of the saving rate, s_w, of the working class. The Pasinetti model has neither the financial sector nor the capital market. Investment in the capital stock K is financed by the internal saving of the capitalist class, i.e. $\frac{\dot{K}}{K} = s_c r$ (ignoring depreciation), where s_c is the ratio of capitalist saving, S_c, to capitalist income, rK, and r is the rate of return to capital (rentals rate = marginal product of capital)-a function of the capital–labor ratio, $k = K/L$, given unit-homogeneity of the production function. In equilibrium, the capital–labor ratio is constant at k^*, so that $\frac{\dot{K}}{K}* = s_c r^* = \frac{\dot{L}}{L}* = n$, or $r^* = \frac{n}{s_c}$ (Pasinetti paradox, absence of s_w). There is no financial intermediary mobilizing bank deposits that can be loaned out to the capitalist class for investment in K. The absence of a capital market also precludes workers from buying the capitalist class' new issues of stocks and bonds to finance investment.

The current paper's model is a neoclassical version of the Pasinetti two-class model that includes a rudimentary financial sector and endogenous learning-by-doing *à la* Villanueva (1994). It generalizes the S-S growth model by introducing a financial intermediary sector and endogenizing labor-augmenting technical change. There are, of course, studies linking finance to output growth. For example, the Atje–Jovanovic (1993) model used an augmented MRW (Mankiw *et al.*, 1992) growth format, introducing finance in the form of a stock market as third input in the aggregate production function, additional to physical capital and effective labor. The model of the current paper views the financial sector as an intermediary that mobilizes workers' financial savings to be lent to the capitalists for investments in the physical capital stock, thus retaining the S-S growth model with the traditional two-input aggregate production function (capital and effective labor).

A new theoretical result is that the equilibrium rate of return to capital is a function of all the structural parameters of the growth model, including both saving rates of the capitalist and working classes. Another major result is that a financial

[1] Although it incorporates rudimentary finance, this paper's model is a special, deterministic version of the more general evolutionary real growth model of Conlisk (1989). See Conlisk (1996) for a closely related contribution to bounded rationality.

crisis, such as happened in 2007-2008 globally and regionally in Asia, produces a short-run recessionary overshooting of a permanently lower equilibrium economic growth rate.

Table 2 organizes the closed economy model, showing group incomes and expenditures (summary income statements). Table 3 shows the summary of sectoral and national balance sheets. The model economy comprises two groups: the capitalist class (*c*) and the working class (*w*). Capitalists own the entire capital stock *K*. I assume that the only financial institution in the model economy is a banking system, and for simplicity, there is no capital market. The capitalist class owns all banks, and the only financial asset available to the working class is a bank deposit. There is no central bank—thus, no currency and no reserve requirements.[2] Investment undertaken by capitalists is financed by internal saving s_cY_c and bank credit *B* (Table 3, Balance Sheet A). Banks extend credit *B* and accept bank deposits *D* (Balance Sheet B). The consolidated balance sheet of the capitalists and banks is shown in Balance Sheet C. When consolidated, net financial flows going to the capitalist class are assumed to be demand-determined by workers' demand for bank deposits *D* (Balance Sheet *D*). Losses on bank credit suffered by banks are absorbed by the capitalist class. Workers are assumed to rent houses or apartments from the capitalists, who hire workers, pay competitive wages, and credit their bank accounts on payday. Bank deposits, insured by banks, are used to write checks or to honor debit cards used as payments for current expenditures. Check-writing and use of debit cards are free of charge and, as offsets, bank deposits pay no interest. For the economy as a whole, the asset (net worth) is the capital stock *K* (Table 3, Balance Sheet E).

Table 2.
Group Incomes and Expenditures

K = capital owned by the capitalist class, *L* (efficiency units) = labor = *AN*, *A* = technology or labor productivity multiplier, *N* = working population, Y_c = income of the capitalist class, Y_w = income of the working class; *r* = rentals rate, *w* = real wage rate, *I* = gross investment, *B* = bank credit, *D* = bank deposits; s_c = saving rate of the capitalist class; s_w = saving rate of the working class; *L* in efficiency units means that if a 2018 man-hour is equivalent as an input in the production function to two man-hours in the base period, say, 2000, then the ratio *K/L* is the amount of capital per half-hour 2018 or per man-hour 2000.

	Capitalists (*c*)	**Workers (*w*)**
Income (*GDP*)	$Y_c = rK$	$Y_w = wL$
Investment in *K*	$I = s_c Y_c + \dot{B}$	
Increment in *B*		$\dot{B} = \dot{D}$
Increment in *D*		$\dot{D} = s_w Y_w$

[2] Although there is no currency, I allow debit cards issued by banks. Bank deposits are necessary to pay credit card charges. In Villanueva (2008, Ch. 7), I include an inflation-targeting, currency-issuing central bank that employs all the traditional monetary policy tools. Fiscal policy and borrowing from global capital markets are covered by Otani and Villanueva (1989) and Villanueva and Mariano (2007, Ch. 6). An endogenous saving rate in a closed economy Ramsey (1928) optimal control set-up is derived by Villanueva (2008, Ch. 5, Appendix 5.B) and in an open economy by Villanueva and Mariano (2021).

Table 3.
Balance Sheets

This table has balance sheet details.

ASSETS	LIABILITIES	NET WORTH
	Panel A. Capitalists	
K	*B*	*K-B*
	Panel B. Banks	
B	*D*	*B-D*
	Panel C. Capitalists/Banks	
K	*D*	*K-D*
	Panel D. Workers	
D	0	*D*
	Panel E. Economy	
K	0	*K*

As Tables 2 and 3 show, external finance of gross investment is constrained by the working class' flow demand for bank deposits. There is no risk of bank runs because bank deposits represent workers' wages that are kept with banks, used by workers to carry out daily transactions. With the aid of this extended and modified model, I analyze the role of finance in the cyclical and in the steady state behavior of economic growth in a non-stochastic environment. To analyze the short-run dynamics of the model, I start from an initial, pre-crisis steady-state GDP growth rate of the model economy. An exogenous crisis (an event outside the model) severely reduces the economy's financial flows, i.e., workers' income and bank deposits, disrupting investments. Two main analytical results are: (1) in the transitional adjustment to the next steady state growth path, the model clearly shows contractionary overshooting of the post crisis steady state growth rate; and (2) absent adequate policy responses, the post-crisis steady state growth path is unambiguously below the pre-crisis one. I show that the recessionary overshooting is followed by a protracted, slow growth recovery, and ultimately to a return to a lower, post-crisis steady-state growth rate.

As simple as this neoclassical growth model is, it can nonetheless account for the broad contours and dynamics of growth developments in the U.S. economy since the financial crisis of 2007-2008. That the growth slowdown has been deep (contractionary overshooting) and the recovery slow are predicted by the model. The model also suggests that countercyclical policies undertaken in response to the crisis may have been insufficient to restore the pre-crisis steady state output growth rate.[3]

Section II presents and discusses the model, followed by the analytics of the reduced model in Section III. Section IV analyzes the short-run (transitional) and long-run (steady state) growth impacts of an exogenous collapse in financial flows. Section V discusses the growth dynamics of changes in the other structural parameters of the model. Section VI summarizes and draws implications for growth policies.

[3] Fiscal policies (income taxes net of benefits, τ_i as fractions of gross income) can easily be incorporated in the model by adopting disposable group incomes $(1-\tau_i)Y_i$, $i = c, w$. Benefits include subsidies and tax credits.

II. THE GROWTH MODEL

$$Y = K^{\alpha} L^{(1-\alpha)} \tag{1}$$

$$r = \frac{\partial Y}{\partial K} \tag{2}$$

$$w = \frac{\partial Y}{\partial L} \tag{3}$$

$$Y_c = rK \tag{4}$$

$$Y_w = wL \tag{5}$$

$$\dot{K} = I - \delta K \tag{6}$$

$$I = s_c Y_c + \dot{B} \tag{7}$$

$$\dot{B} = \dot{D} \tag{8}$$

$$D = s_w Y_w \tag{9}$$

$$L = AN \tag{10}$$

$$\frac{\dot{A}}{A} = \emptyset \frac{K}{L} + \lambda \tag{11}$$

$$\frac{\dot{N}}{N} = n \tag{12}$$

$$k = \frac{K}{L} \tag{13}$$

$$d = \frac{D}{L} \tag{14}$$

where $Y = GDP$, K = capital stock, L = effective labor, r = rentals rate, w = real wage rate, Y_c = capitalist income, Y_w = worker income, I = gross investment, B = bank credit, D = bank deposits, A = technology/productivity multiplier, N = exogenous working population, k = ratio of K to L, d = ratio of D to L, and α, s_c, s_w, $\emptyset$, δ, λ and n = structural parameters.

Equation (1) is the aggregate production function, subject to the Inada (1963) conditions.[4] Equations (2) and (3) are profit-maximizing conditions, setting the rental rate and real wage rate equal to their respective marginal products. Equations (4) and (5) define group incomes. Equation (6) states that the addition to the capital

[4] With reference to any production function, $Y=F(K,L)=Lf(k)$, where K is capital, L is labor, and k is the ratio of K to L, these conditions can be summarized as follows: $\lim \frac{\partial F}{\partial K} = \infty$ as $K \to 0$; $\lim \frac{\partial F}{\partial K} = 0$ as $K \to \infty$; $f(0) \geq 0$; $f'(k) > 0$, and $f''(k) < 0$ for all $k > 0$. Equation (1) satisfies these conditions.

stock equals gross investment less depreciation. Equation (7) says that gross investment is financed by capitalists' own internal saving, which is a proportion s_c of their income, and by bank credit. Equations (8) and (9) state that the flow of bank credit is financed by bank deposits, representing a proportion s_w of workers' income after paying for consumption and other current expenditures. Equation (10) defines effective[5] labor L as the product of labor-augmenting technology or productivity multiplier A and working population N.[6] Equation (11) postulates that the increment in labor-augmenting productivity has endogenous and exogenous components. The endogenous element is a modified version of Arrow's (1962) learning by doing. Arrow (1962) hypothesizes that investment ($\dot{K}$) induces more learning-by-doing, $\frac{\dot{A}}{A} = \emptyset\left(\frac{\dot{K}}{K}\right) + \lambda$, i.e., learning-by-doing is a proportion $\emptyset$ of capital growth plus an exogenous component λ.[7] In equilibrium, Arrow's output growth is exogenously fixed at $(\frac{\dot{K}}{K})^{*} = (\frac{\dot{L}}{L})^{*} = (\frac{\dot{A}}{A})^{*} + n = \left(\frac{\dot{Y}}{Y}\right)^{*} = g^{Y^{*}} = \frac{(\lambda+n)}{(1-\emptyset)}$, which is a multiple of the S-S equilibrium output growth since $0<\emptyset<1$ by assumption. If Arrow's learning-by-doing is interpreted as driven by cumulative investments (integral of I, i.e., stock of K) per unit of L, Equation (11) adopts, not modifies, Arrow's learning-by-doing.[8] The larger the capital stock per capita, the more intense is the workers' learning-by-doing experience and, hence, the higher is their productivity.[9] The exogenous component is the standard S-S (and Arrow's) constant term λ. Equation (12) assumes a constant rate n of exogenous population growth. The remaining Equations (13)-(14) are definitions for capital intensity k, and deposit ratio d, respectively.

There are 14 equations and 14 variables and time t (suppressed): $Y, K, L, r, w, Y_c, Y_w, D, B, I, A, N, k,$ and d.[10] The growth model reduces to two differential equations in the state variables k and d. If $s_w = 0$ and $\emptyset = 0$ (no financial sector and all technical change is exogenous), Equations (8) and (9) drop out, Equation (7) becomes $I=sY$ (s is the aggregate saving rate) and the growth model reduces to the S-S model. If $s_w = 0$ and $\emptyset > 0$ (no financial sector and technical change is partly endogenous via learning-by-doing), the model reduces to the Villanueva (1994) model.

III. REDUCED MODEL

Using Equations (4), (5), (8) and (9), Equation (7) becomes:

$$I = s_c rK + s_w wL \tag{15}$$

5 In efficiency units (for definition, see top of Table 2).

6 Generally, the definition of L should be $L = APN$, where P is the labor participation rate ($0 < P \leq 1$). The working population is PN. When $P = 1$, $L = AN$. Whatever P is, it is usually assumed in growth literature as an exogenous constant, whose rate of change is zero. For an endogenous and variable P, see Villanueva (2020).

7 $\emptyset$ is a learning coefficient.

8 The only difference is the absence of a financial sector in the Arrow model.

9 Using $L = AN$ and $k = K/L$, re-write Equation (11) as $\dot{A} = \emptyset\frac{K}{N} + \lambda A$.

10 Time is not explicit in the model. Partly for this reason, phase-diagramming is used to analyze the existence, uniqueness, and stability of the growth model.

Equation (15) into Equation (6), using Equations (1)-(5) and (8) and (9), and dividing the result by K,

$$\frac{\dot{K}}{K} = [s_c\alpha + s_w(1-\alpha)]k^{(\alpha-1)} - \delta \tag{16}$$

Equation (16) is the warranted rate equation.

Equations (10)-(12) need elaboration. The differential of Equation (10) is given by:

$$\dot{L} = \dot{A}N + A\dot{N}$$

The increment in L is the sum of the increment in labor-augmenting technical change, specified in Equation (11), and the increment in the working population, specified in Equation (12). Equation (11) can be re-written as $\dot{A} = \emptyset\left(\frac{K}{N}\right) + \lambda A$ (refer back to footnote 9). It is the size of the capital stock *per capita* that matters for learning-by- doing, not the growth rate of the capital stock. When laborers work with a larger capital stock possessing the latest advanced technology, they learn more as time passes, with consequent increase in productivity. Equation (11) is identical to the specification in Villanueva (1994). The model allows for a constant rate λ of exogenous labor-augmenting technical change as in the S-S growth model.

Time differentiating Equation (10) and substituting Equations (11)-(12),

$$\frac{\dot{L}}{L} = \emptyset k + \lambda + n \tag{17}$$

Equation (17) is the natural rate equation.

Time differentiating Equation (13) and substituting into the result Equations (16)- (17) yields the proportionate rate of change in the capital intensity,

$$\frac{\dot{k}}{k} = [s_c\alpha + s_w(1-\alpha)]k^{(\alpha-1)} - \emptyset k - (\lambda + n + \delta). \tag{18}$$

From Equations (1) and (13), $Y=Lk^{\alpha}$, whose time derivative is

$$\frac{\dot{Y}}{Y} = \frac{\dot{L}}{L} + \alpha\frac{\dot{k}}{k} \tag{19}$$

Equation (19) is the growth rate of output at any moment of time. Using Equation (17) evaluated at the steady state $k = k^*$[11], Equation (19) can be re-written as

$$\frac{\dot{Y}}{Y} - n = \emptyset k^* + \lambda + \alpha\frac{\dot{k}}{k} \tag{20}$$

[11] A constant k^* implies $\frac{\dot{K}}{K}* = \frac{\dot{L}}{L}* = \frac{\dot{Y}}{Y}* = \emptyset k^* + \lambda + n$. This balanced growth path is a consequence of the unit-homogeneous production function.

where $\frac{\dot{k}}{k}$ is given by Equation (18). Equation (20) states that the growth rate of per capita output $\frac{\dot{Y}}{Y} - n$ will be above (below) the steady state level $=\emptyset k^{*}+\lambda$ for a rising (falling) k, or whenever k is smaller (larger) than k^{*}.[12]

Finally, time differentiating Equation (14) and substituting Equations (1), (3), (5), (9) and (17) yield

$$\frac{\dot{d}}{d} = \frac{\{s_w(1-\alpha)\}k^{\alpha}}{d} - (\emptyset k \ + n + \lambda) \tag{21}$$

Equations (18) and (21) form the reduced model in k, d, and time. The equilibrium values of k and d, denoted by k^* and d^*, are obtained by equating Equations (18) and (21) to zero, i.e.,

$$[s_c\alpha + s_w(1-\alpha)]k^{*(\alpha-1)} - \emptyset k^{*} - (\lambda \ + \ n + \delta) = 0 = H(k^{*}, d^{*}), \tag{22}$$

$$\frac{\{s_w(1-\alpha)\}k*^{\alpha}}{d*} - (\emptyset k * \ + n + \lambda) = 0 = J(k^{*}, d^{*}), \tag{23}$$

whose partial derivatives are:

$$a_{11} = \frac{\partial \frac{\dot{k}}{k}}{\partial k} = H_1 = [s_c\alpha + s_w(1-\alpha)](\alpha - 1)k^{*(\alpha-2)} - \emptyset < 0 \tag{24}$$

$$a_{12} = \frac{\partial \frac{\dot{k}}{k}}{\partial d} = H_2 = 0 \tag{25}$$

$$a_{21} = \frac{\partial \frac{\dot{d}}{d}}{\partial k} = J_1 = s_w(1-\alpha)\alpha k^{*(\alpha-1)} - \emptyset > 0 \text{ [13]} \tag{26}$$

$$a_{22} = \frac{\partial \frac{\dot{d}}{d}}{\partial d} = J_2 = -d^{*-2}s_w(1-\alpha)k^{*\alpha} \ < 0 \tag{27}$$

Denoting the Jacobian matrix as A, whose elements are the a_{ij} above, the stability conditions are:

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[12] It can also be seen from Equation (20) that the equilibrium growth rate of per capita output $\frac{\dot{Y}}{Y} * -n \ = \emptyset k^{*} + \lambda$, since the third term on the right-hand side disappears when the economy reaches equilibrium at $\frac{\dot{k}}{k} = 0$.

[13] The inequality in Equation (26) is a reasonable approximation. The average capital services to employment ratio is 3 (1987-2020, www.bls.gov) and the average annual working population growth rate is 1 percent (1994-2014, www.bls.gov). The average personal saving rate is 10 percent (www.bea.gov) and the capital share $\alpha = 0.3$. Out of a hypothetical 3 percent economic growth rate, an assumed value of 0.003 for the learning coefficient $\emptyset$ adds 0.9 percentage point due to learning-by-doing, 1 percentage point to working population growth, and (residually) 1.1 percentage point due to exogenous technical change. The value of $\emptyset$=0.003 was used by Villanueva (1994, 2008), with $k^* = 3$.

tr $(A) < 0$

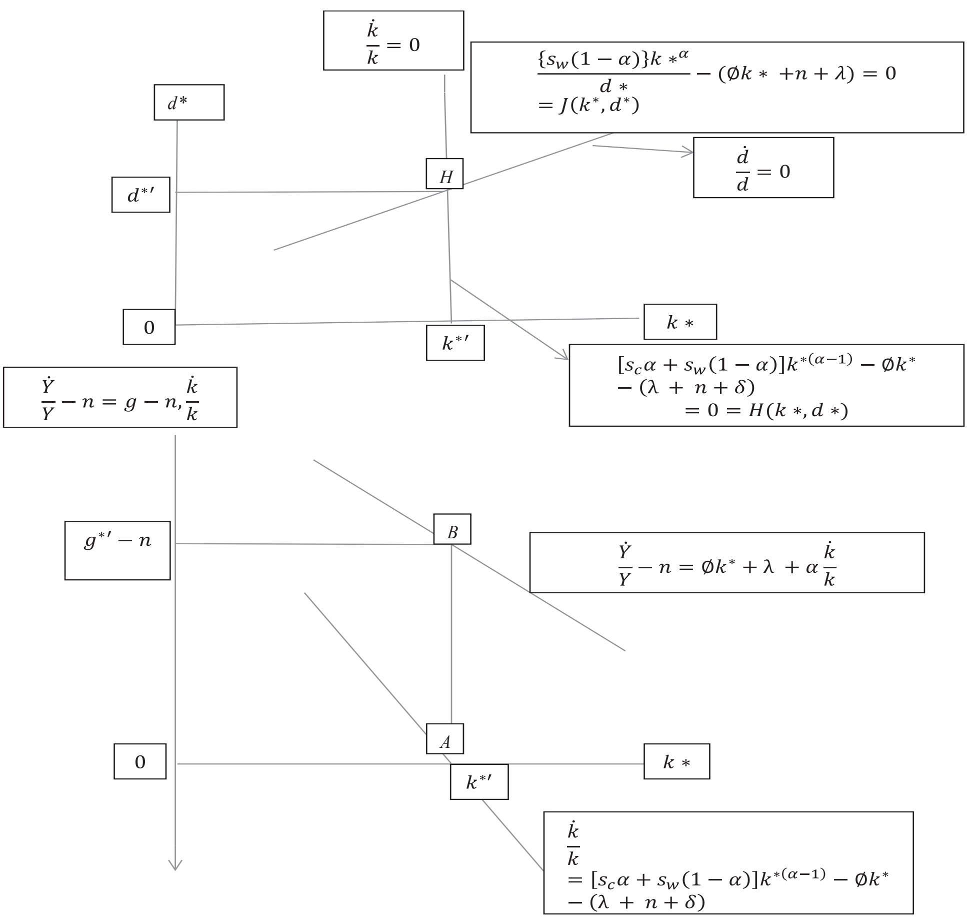
and

$|A| > 0$

are met, given Equations (24)-(27). Thus, the model is stable in the neighborhood of the steady state.

The phase diagram of the reduced model is shown in Figure 1, showing long-run equilibrium.[14] In both the upper and lower parts, the horizontal axis measures the capital intensity k. In the upper part, the vertical axis measures bank deposits per effective worker, d, while in the lower part, it measures the proportional rates of change in per capita *GDP* and capital intensity.

Figure 1.
Long-run Equilibrium



[14] A phase diagram is a graphical tool to analyze the existence and stability of equilibrium for a system of two first-order differential equations not explicitly involving time.

The upper part of Figure 1 plots the $\frac{\dot{k}}{k}=0$ and $\frac{\dot{d}}{d}=0$ lines in the *k, d* space. In Equation (18), the $\frac{\dot{k}}{k}$ relationship does not involve *d*, so that the $\frac{\dot{k}}{k}=0$ curve is a straight line from the steady state value of *k* at $k^{*\prime}$. In Equation (21), the $\frac{\dot{d}}{d}=0$ line is upward-sloping because as *k* goes up, the real wage rises, stimulating higher demand for bank deposits. The long-run equilibrium values of $k = k^{*\prime}$ and $d = d^{*\prime}$ are obtained when $\frac{\dot{k}}{k}=0$ and $\frac{\dot{d}}{d}=0$ lines intersect at point *H*, which is locally stable. Starting from any point other than at point *H* in the upper part of Figure 1, *k* and *d* will go back to $k^{*\prime}$ and $d^{*\prime}$, respectively. For example, any point to the right of the $\frac{\dot{k}}{k}=0$ line indicates $k > k^{*\prime}$, $(\frac{\dot{k}}{k}<0)$, meaning that the natural rate exceeds the warranted rate. For balanced growth to be restored, the warranted rate must rise and the natural rate must fall. The marginal returns to capital increases, encouraging investment, and accelerating the warranted rate. As *k* falls toward $k^{*\prime}$, learning-by-doing slows, decelerating the natural rate. Consequently, $\frac{\dot{k}}{k}$ will be less and less negative, until it is zero at equilibrium point H, when the warranted rate has risen to the lower natural rate restoring balanced growth at $k^{*\prime}$ $(\frac{\dot{k}}{k}=0)$. The opposite sequence of economic events holds at any point to the left of the $\frac{\dot{k}}{k}=0$ line when $k < k^{*\prime}$ $(\frac{\dot{k}}{k}>0)$.

The $\frac{\dot{d}}{d}=0$ line is upward-sloping because the inequality in Equation (26) says that the positive income effect of a higher *k* on $\frac{\dot{D}}{D}$ is greater than the positive learning-by-doing effect of a higher k^* on $\frac{\dot{L}}{L}$. Take any point below the $\frac{\dot{d}}{d}=0$ line. To the right of $k^{*\prime}$ ($k^* > k^{*\prime}$), the higher income effect on $\frac{\dot{D}}{D}$ is larger than the higher learning-by-doing effect; *d* goes up. To the left of $k^{*\prime}$ ($k^* < k^{*\prime}$), as k^* goes up toward $k^{*\prime}$, the higher income effect on $\frac{\dot{D}}{D}$ is larger than the higher learning-by-doing effect; *d* goes up. Now, take any point above the $\frac{\dot{d}}{d}=0$ line. To the left of $k^{*\prime}$ ($k^* < k^{*\prime}$), the lower income effect on $\frac{\dot{D}}{D}$ is larger than the lower learning-by-doing effect; *d* goes down. To the right of $k^{*\prime}$ ($k^* > k^{*\prime}$), as k^* goes down toward $k^{*\prime}$, the lower income effect on $\frac{\dot{D}}{D}$ is larger than the lower learning-by-doing effect; *d* goes down.

The lower panel of Figure 1 shows the instantaneous and steady state per capita *GDP* growth rate. Taking the derivatives of the $\frac{\dot{Y}}{Y}-n$ and $\frac{\dot{k}}{k}$ lines with respect to *k*, the $\frac{\dot{Y}}{Y}-n$ line is drawn flatter than the $\frac{\dot{k}}{k}$ line.[15] At $k^{*\prime}$, the instantaneous and steady state per capita *GDP* growth rate coincide at $g^{*\prime}$-n. At any $k < k^{*\prime}$, $\frac{\dot{k}}{k}>0$, and the instantaneous per capita *GDP* growth rate exceeds the steady state rate $(\frac{\dot{Y}}{Y}-n>g^{*\prime}-n)$. At any $k > k^{*\prime}$, $\frac{\dot{k}}{k}<0$, and the instantaneous per capita *GDP* growth rate is below the steady state rate $(\frac{\dot{Y}}{Y}-n>g^{*\prime}-n)$. Through adjustments

15 $\frac{\partial(\frac{\dot{Y}}{Y}-n)}{\partial k}=\frac{\alpha\partial\left(\frac{\dot{k}}{k}\right)}{\partial k}<\frac{\partial(\frac{\dot{k}}{k})}{\partial k}$ in absolute value, since α is a positive fraction.

in the capital intensity and its rate of change, the warranted and natural rates adjust towards equality to restore a balanced growth path of per capita output at the rate $\emptyset k^{*\prime} + \lambda$ (at $\frac{\dot{k}}{k} = 0$).

IV. SHORT-RUN AND LONG-RUN GROWTH EFFECTS OF A DECLINE IN FINANCIAL FLOWS

Figure 2 reproduces Figure 1. It traces the dynamic effects of a decline in the proportion s_w of workers' income so that there is a decrease in net financial flows (from s_{w0} to s_{w1}) the capitalist/financial intermediary sector (as happened in the financial crisis of 2007-2008). The initial steady state equilibrium points are $A(k^{*\prime},0)$ and $C(k^{*\prime}, g^{*\prime} - n)$, where $g^{*\prime} - n$ and $k^{*\prime}$, respectively, denote equilibrium growth rate of per capita output and equilibrium level of capital intensity. Given $k^{*\prime}$, the initial equilibrium deposit ratio is $d^{*\prime}$ in the upper panel. A lower s_w (s_{w1}) shifts downward both the $\frac{\dot{d}}{d}$ line in the upper panel and the $\frac{\dot{Y}}{Y} - n$ line in the lower panel of Figure 2. Both steady state deposit-labor ratio and capital intensity are lower at $d^{*\prime\prime}$ and $k^{*\prime\prime}$.[16] In the lower panel, the $\frac{\dot{k}}{k}$ line shifts downward to the left and intersects the k-axis at point $B(k^{*\prime}, 0)$. The $\frac{\dot{Y}}{Y} - n$ line also shifts downward to the left, and its new steady state equilibrium point is $D(k^{*\prime\prime}, g^{*\prime\prime} - n)$.

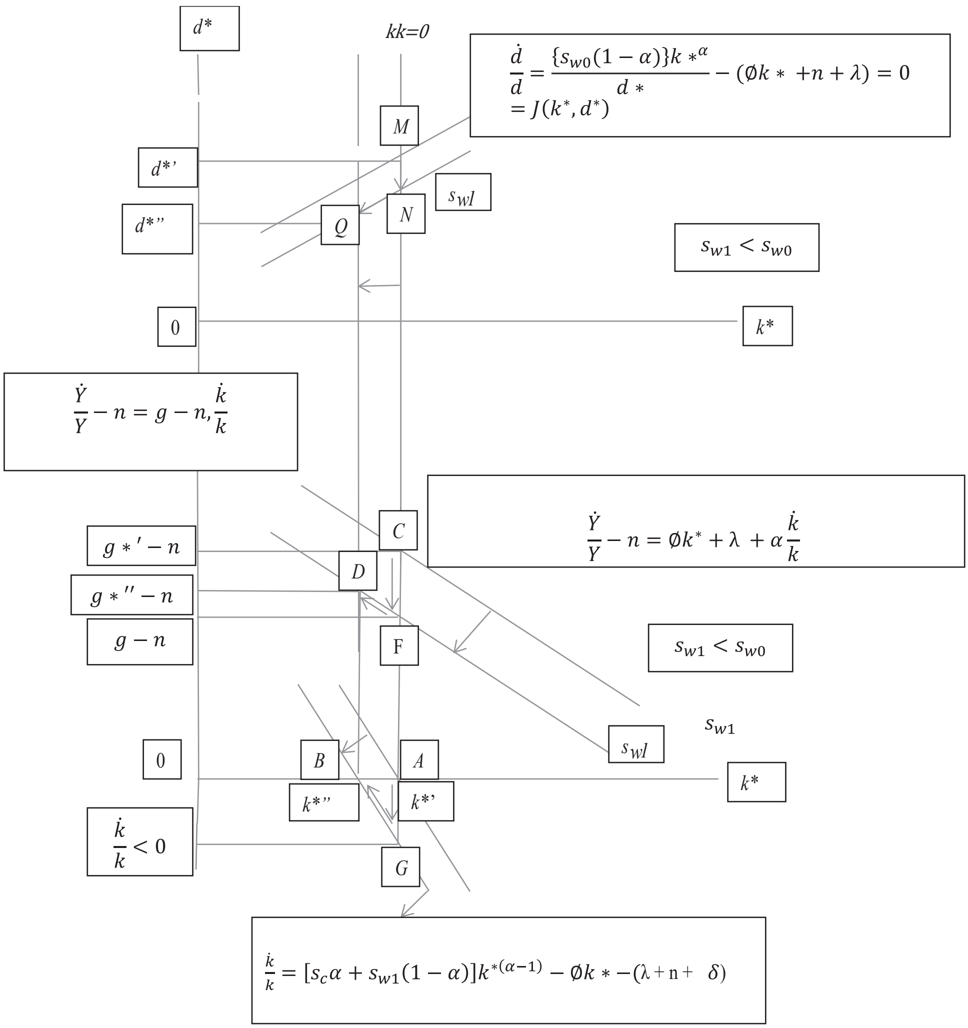
The short-run growth dynamics is as follows. Following the downward shift of the $\frac{\dot{k}}{k}$ line under the impact of a lower s_{w1}, and before the initial steady-state $k = k^*$ adjusts, $\frac{\dot{k}}{k} < 0$ at point G. Thus, the per capita output growth rate drops precipitously from $g^{*\prime} - n$ at point C to $g - n$ at point F. As $k^* = k^{*\prime}$ contracts toward $k^{*\prime\prime}$, the warranted rate accelerates on the strength of improvements in the marginal and average returns to capital, exceeding the decelerating natural rate, the latter reflecting lower learning-by-doing (the negative value of $\frac{\dot{k}}{k}$ becomes less and less negative) until the two rates are equal at the new steady state equilibrium point $B(k^{*\prime\prime},0)$[17]. Correspondingly, the per capita output growth rate recovers from $g - n$ at F and increases to $g^{*\prime\prime} - n$ at D on the strength of a rising last term $\frac{\dot{k}}{k}$ of the $\frac{\dot{Y}}{Y} - n$ line until this term is zero at D.[18] As for the dynamics of the adjustment of deposits per effective worker, d^* initially declines from point M to point N in the upper panel, and further falls from point N to the new equilibrium point Q because of lower real wages as capital intensity shrinks.

[16] Recall that in the Pasinetti (1962) growth model, the equilibrium rate of return to capital r^* (capital's marginal product) is inversely related to the capitalists' saving rate, but is unaffected by the workers' saving rate. This result carries over in the modified Pasinetti model with endogenous growth discussed in this paper (see Table 1). The additional new result is that r^* is also inversely related to the workers' saving rate. As workers decrease their financial savings, deposit flows into the financial system fall, capitalists' physical investment goes down and capital intensity declines, raising the equilibrium rate of return to capital r^*.

[17] Adjustment is traced by segment *GB*.

[18] Adjustment is traced by segment *FD*.

Figure 2.
Short-run and Long-run Effects of a Decline in Net Financial Flows (Lower s_w) on the Growth Rate of Per Capita Output/Income



The 2007-2008 global financial crisis precipitated by the U.S. financial crisis can be analyzed with the help of Figure 2. Following the financial shock (represented by the downward shift in s_w), credit flows to the corporate sector dried up, investments slumped, leading to a short-run precipitous drop in the growth rate of per capita output. As time went on, the recovery was excruciatingly slow and after adjustments finished, the equilibrium per capita output growth remained lower than the pre-crisis rate. The pre-crisis output growth rate at $g^{*\prime}$ - n could have been achieved with quantitatively sufficient, calibrated expansionary fiscal and monetary policies promoting investments.

V. OTHER COMPARATIVE DYNAMICS

Table 1 shows the all-else-equal effects of the model's other structural parameters on equilibrium capital intensity and equilibrium per capita output growth.

$$\frac{\dot{k}}{k} = [s_c\alpha + s_w(1-\alpha)]k^{(\alpha-1)} - \emptyset k - (\lambda + n + \delta). \tag{18}$$

$$\frac{\dot{Y}}{Y} - n = \emptyset k^* + \lambda + \mathrm{n} + \alpha\frac{\dot{k}}{k}. \tag{20}$$

Table 1.
Sensitivity of k^*, r^*, and g^* - n to Parameter Changes

The $(k)^{\cdot}/k$=0 [Equation (22)] is the steady-state condition with an implicit solution for k^*. The reader can easily check the signs of Table 1 by implicit differentiation of k^* in the equation $\frac{\dot{k}}{k} = 0$ with respect to the parameters. Substituting such changes in k^* into the partial derivatives of $r^* = \alpha k^{*(\alpha-1)}$ and of $\frac{\dot{K}}{K} = 0 = \frac{\dot{L}}{L} = 0 = \frac{\dot{Y}}{Y} = 0$ [Equations (16)-(19), evaluated at k^*] with respect to each of the 6 structural parameters determines the signs of the partial derivatives shown in Table 1. k^* = capital intensity, r^* = rate of return to capital (marginal product of capital), g^* - n = per capita output growth, s_c = capitalist saving rate, s_w = worker saving rate, $\emptyset$ = learning coefficient, n = rate of exogenous population growth, λ = rate of exogenous labor-augmenting technical change, and δ = rate of capital depreciation. * indicates steady-state (equilibrium) values.

Description	s_c	s_w	$\emptyset$	n	λ	δ
Change in k^*	+	+	-	-	-	-
Change in r^*	-	-	+	+	+	+
Change in g^* - n	+	+	+	-	+	-

The signs in Table 1 are obtained by equating Equation (18) to zero and partially differentiating with respect to each structural parameter, noting that $k^* = k^*(s_c, s_w, \emptyset, n, \lambda, \delta)$. For example, to find the sign of $\frac{\partial k^*}{\partial s_w}$, which is the effect of a fall in the saving rate of the working class:

$$(1-\alpha)k^{*(\alpha-1)} + \{[s_c\alpha + s_w(1-\alpha)](\alpha-1)k^{*(\alpha-2)} - \emptyset\}\frac{\partial k^*}{\partial s_w} = 0$$

or,

$$\frac{\partial k^*}{\partial s_w} = \frac{-(1-\alpha)k^{*(1-\alpha)}}{\{[s_c\alpha + s_w(1-\alpha)](\alpha-1)k^{*(\alpha-2)} - \emptyset\}} > 0, \text{ since } 0 < \alpha < 1.$$

A decline in the saving rate of the working class leads to a fall in equilibrium capital intensity, inducing lower learning-by-doing, and, hence, lower equilibrium per capita output growth, $\frac{\dot{Y}^*}{Y} - n = \emptyset k^* + \lambda$ (Equation (20)).

Figure 3 illustrates the growth effects of changes in the learning coefficient $\emptyset$. The initial equilibrium occurs at points $A(k^{*\prime},0), B(k^{*\prime},g^{*\prime}\text{-}n)$, and the subsequent equilibrium is at points $C(k^{*\prime\prime},0), D(k^{*\prime},g^{*\prime\prime}-n)$. Following the downward shift of the $\frac{\dot{k}}{k}$ line under the impact of a higher $\emptyset$, and before the initial steady-state $k = k^{*\prime}$ adjusts, $\frac{\dot{k}}{k} < 0$ at point G. Thus, per capita output growth rises only to $g - n$ at point E, less than $g^{*\prime\prime} - n$ at point D. As $k = k^{*\prime}$ contracts toward $k^{*\prime\prime}$, the warranted rate accelerates on the strength of improvements in the marginal and average returns to capital, exceeding the decelerating natural rate, the latter reflecting lower learning-by-doing (the negative value of $\frac{\dot{k}}{k}$ at point *G* becomes less and less negative) until the two rates are equal at the new steady state equilibrium point *C(k*′′,0)*[19]. Correspondingly, the per capita output growth rate rises further from *g - n* at *E* to $g^{*\prime\prime}$ - *n* at *D* on the strength of a rising last term $\frac{\dot{k}}{k}$ of the $\frac{\dot{Y}}{Y} - n$ line until this term is zero at *D*.[20]

Figure 3.
Short-run and Long-run Effects of an Increase in the Learning Coefficient (Higher ∅) on the Growth Rate of Per Capita Output/Income

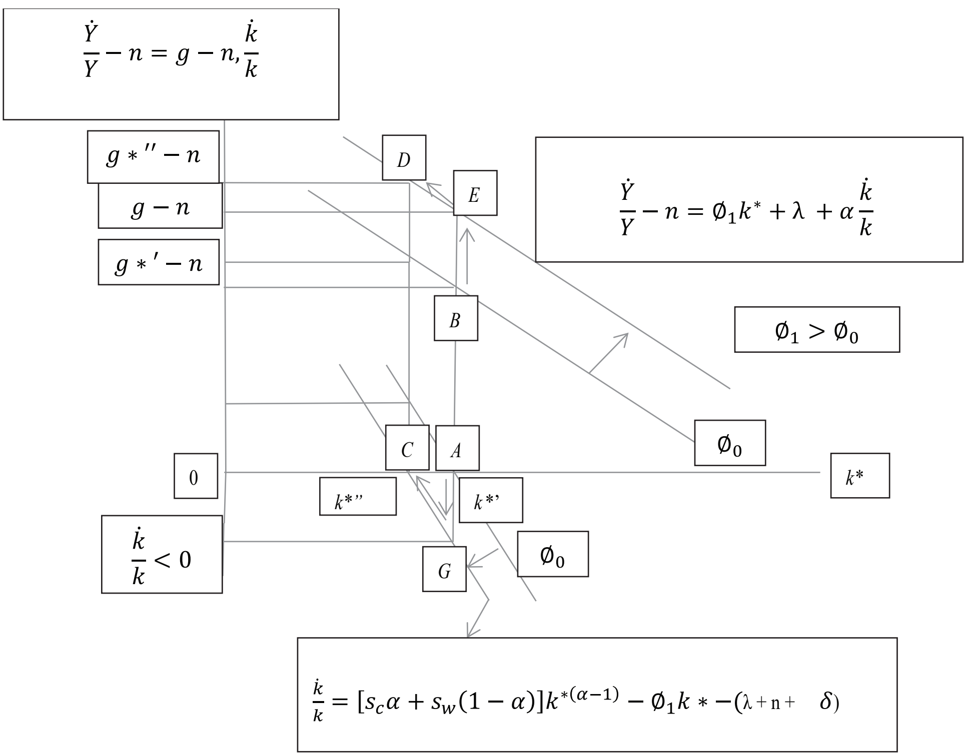
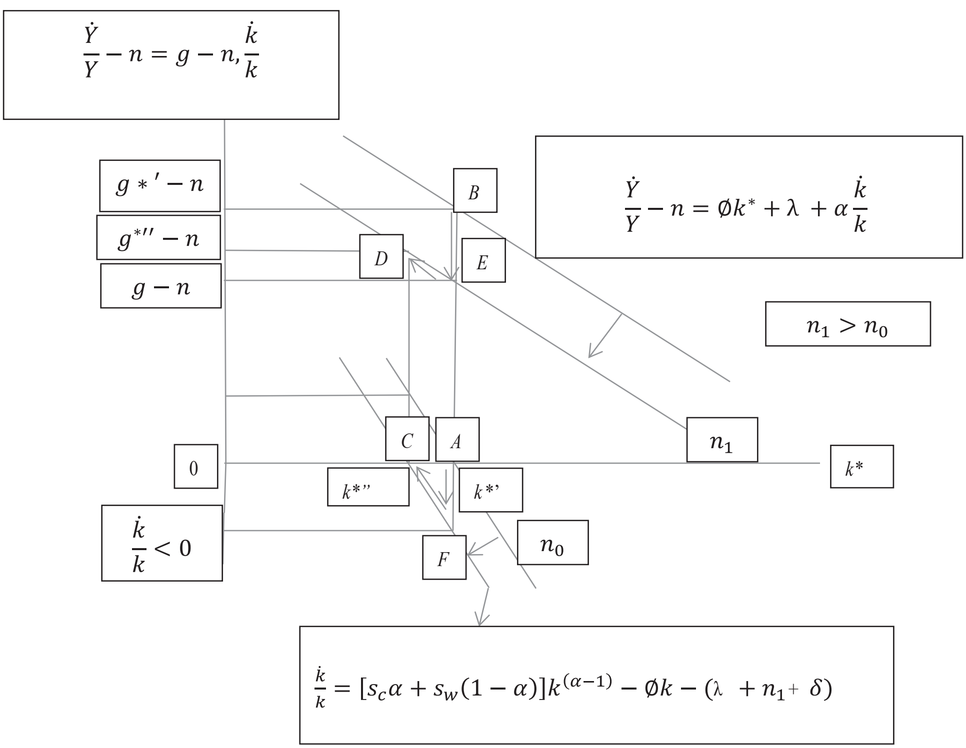


Figure 4 shows the growth effects of an increase in population growth *n*. The initial equilibrium occurs at points *A(k*′,0),B(k*′,g*′ - n)*, and the subsequent equilibrium is at points *C(k*′′,0),D(k*′′,g*′′ – n)*, characterized by lower capital

[19] Adjustment is traced by segment *GC*.
[20] Adjustment is traced by segment *ED*.

intensity and lower per capita output growth.[21] Following the downward shift of the $\frac{\dot{k}}{k}$ line under the impact of a higher n, and before the initial steady-state $k = k^*$ adjusts, $\frac{\dot{k}}{k} < 0$ at point *F*. Thus, per capita output growth drops precipitously to $g - n$ at point *E*, less than $g^{*\prime\prime} - n$ at point *D*. As $k = k^{*\prime}$ contracts toward $k^{*\prime\prime}$, the warranted rate accelerates on the strength of improvements in the marginal and average returns to capital, exceeding the decelerating natural rate, the latter reflecting lower learning-by-doing (the negative value of $\frac{\dot{k}}{k}$ at point *F* becomes less and less negative) until the two rates are equal at the new steady state equilibrium point *C*($k^{*\prime\prime}$,0)[22]. Correspondingly, the per capita output growth rate rises further from g - n at *E* to $g^{*\prime\prime}$ - n at *D* on the strength of a rising last term $\frac{\dot{k}}{k}$ of the $\frac{\dot{Y}}{Y} - n$ line until this term is zero at *D*.[23]

Figure 4.
Short-run and Long-run Effects of an Increase in Population Growth (Higher *n*) on the Growth Rate of Per Capita Output/Income



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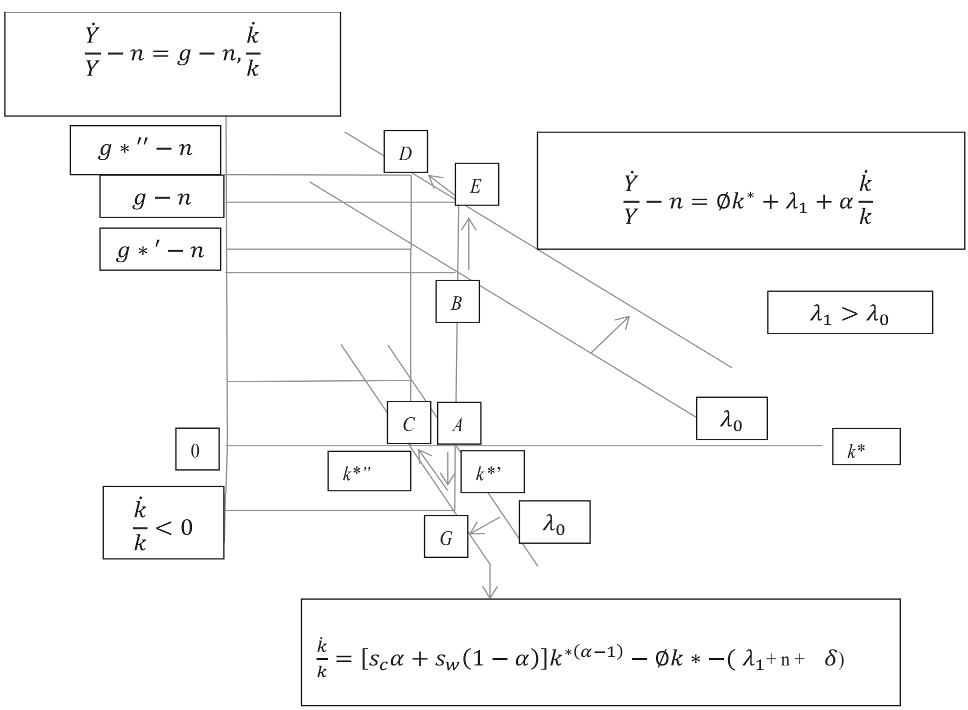
[21] As mentioned earlier, the Arrow (1962) model's steady state growth equation is $\frac{\dot{K}}{K}* = \frac{\dot{L}}{L}* = \frac{\dot{A}}{A}* + n = \frac{\dot{Y}}{Y}* = g^Y* = (\mu + n)/(1-\emptyset)$, which has the property that $\frac{d(g^{Y*}-n)}{dn} = [\frac{\emptyset}{(1-\emptyset)}] > 0$, i.e., an *increase* in the population growth rate n *raises* the long-run growth rate of per capita output, g^{Y*}-n. This prediction is counterintuitive and rejected by empirical evidence. See, among others, Conlisk (1967), Otani and Villanueva (1990), Knight *et al*. (1993), and Villanueva (1994).

[22] Adjustment is traced by segment *FC*.

[23] Adjustment is traced by segment *ED*.

Finally, Figure 5 illustrates the growth effects of an increase in exogenous labor-augmenting technical change λ. The initial equilibrium occurs at points *A(k*′,0),B(k*′,g*′ - n)*, followed by the next equilibrium at points *C(k*′′,0),D(k*′′,g*′′ – n)*, characterized by lower capital intensity and higher per capita output growth. Following the downward shift of the $\frac{\dot{k}}{k}$ line under the impact of a higher λ, and before the initial steady-state $k = k^{*\prime}$ adjusts, $\frac{\dot{k}}{k} < 0$ at point *G*. Thus, per capita output growth rises only to $g - n$ at point *E*, less than $g^{*\prime\prime} - n$ at point *D*. As $k = k^{*\prime}$ contracts toward $k^{*\prime\prime}$, the warranted rate accelerates on the strength of improvements in the marginal and average returns to capital, exceeding the decelerating natural rate, the latter reflecting lower learning-by- doing (the negative value of $\frac{\dot{k}}{k}$ at point *G* becomes less and less negative) until the two rates are equal at the new steady state equilibrium point *C(k*′′,0)*.[24] Correspondingly, the per capita output growth rate rises further from g - n at *E* to $g^{*\prime}$ - n at *D* on the strength of a rising last term $\frac{\dot{k}}{k}$ of the $\frac{\dot{Y}}{Y} - n$ line until this term is zero at *D*.[25]

Figure 5.
Short-run and Long-run Effects of an Increase in Exogenous Labor-augmenting Technical Change (Higher λ) on the Growth Rate of Per Capita Output/Income



[24] The adjustment is traced by segment *GC*.
[25] The adjustment is traced by segment *ED*.

Table 1 suggests a regression format to test the long-run growth implications of the model's structural parameters using cross-country data. The saving rates of the capitalist and working classes, population growth, and depreciation rates (all observables) can be regressors. The rate of exogenous labor-augmenting technical change can be impounded in the constant term of the regression, and the learning coefficient can be replaced by its determinants as was done by Villanueva (1994), i.e., expenditures on education and health, and foreign trade (exports plus imports) to GDP.

VI. CONCLUSION

In this paper's modified Pasinetti framework with endogenous growth, the equilibrium rate of return to capital is shown to be a function of all structural parameters, including both saving rates of the capitalist and working classes. Additionally, this paper shows the simple analytics of the short-run and long-run growth effects of an exogenous collapse in financial flows, as happened in the global financial crisis of 2008-2009. In the short run, there is an overshooting of recessionary growth. In the long run, there is an eventual return to a lower output growth path. Policies to restore the pre-crisis growth path involve appropriately calibrated expansionary monetary and fiscal policies, as well as supportive financial regulation to ensure the health of the financial sector. The objective is to ensure adequate financing of the economy's investment and capital accumulation through higher capitalists' re-invested profits and workers' financial savings.

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I am indebted to Lee Endress, Gonzalo Jurado and an anonymous referee for valuable comments.

In my graduate courses in growth theory in the early 1960s, I liked macro better than micro. The reason had nothing to do with level of aggregation, but rather with a difference in approach. In macro at the time, we would write down plausible behavioral relations, phrased as a differential equation system, and let the adaptive dynamics play out. What would happen? What would we learn? The macro approach seemed closer to behavior and more open to novelty and imagination.

Conlisk (2004) on the occasion of a *Festschrift* for Nobel Laureate Herbert A. Simon

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