ESTIMATING A JOINT PROBABILITY OF DEFAULT INDEX FOR INDONESIAN BANKS: A COPULA APPROACH

Zaäfri A. Husodo*, Sigit S. Wibowo**, M. Budi Prasetyo**, and Usman Arief**, Maulana Harris Muhajir***

* Corresponding author. Department of Management, Faculty of Economics and Business, Universitas Indonesia, Depok, Indonesia. Email: z.husodo@ui.ac.id

** Department of Management, Faculty of Economics and Business, Universitas Indonesia, Depok, Indonesia.
***Department of Macroprudential Policy, Bank Indonesia.

ABSTRACT

We develop a joint default probability index to signal potential systemic risks in the highly concentrated Indonesian banking industry. To build the index, we estimate bank-level tail risks using monthly bank financial reports. We use the copula approach to derive the joint multivariate dependencies at the bank level, as reflected in the monthly financial reports. Our results, which are based on a sample of 104 banks from December 2003 to April 2020, show joint multivariate dependencies at the bank level suggesting that the standard univariate normal distribution is unsuitable for capturing tail risks of individual banks. Our index accurately captures the global financial crisis of 2007-2008 indicating that it is a valid joint default probability index. Further, our index also signaled a higher degree of joint default before the COVID-19 outbreak in 2020, suggesting that it is a good indicator of potential systemic risk in the economy.

Keywords: Copula; Pair copula construction; Systemic risk; Financial system. **JEL Classifications: G18; G21; C15.**

Article history:Received: June 24, 2019Revised: August 18, 2019Accepted: September 09, 2020Available online: October 30, 2020https://doi.org/10.21098/bemp.v23i3.1358

I. INTRODUCTION

Central to the issue of financial system stability is quantifying potential systemic risk in the economy. The condition of the financial system determines the direction of the economy (Juhro and Iyke, 2019), and hence, being able to gauge the potential risk to the financial system is important to policymakers. Bisias *et al.* (2012) summarize this issue and emphasize that there is no single "pressure gauge" that can adequately detect crises. The two approaches to estimating systemic risk are supervisory- and market-based approaches. The first relies on firm-specific information using annual accounting and other confidential data provided to regulators by financial institutions and not captured by markets (Basel Committee on Banking Supervision, 2018; Greenwood *et al.*, 2015). The latter relies on publicly available market data, namely stock price and credit default swaps (CDS) (see also Benoit *et al.* (2017)).

Benoit *et al.* (2017) argue that the data needed for the supervisory approach are disclosed with a lag. Conversely, the market-based approach considers high-frequency data with timely information, and, hence, it is more sensitive to changes in systemic risk regimes but prone to noise and estimation bias (Black, 1986; Henker & Husodo, 2010). In some cases, this bias will be amplified to the tail event condition resulting in an upward bias of the estimated risk. There is still an opportunity to quantify potential systemic risk from balancing information accuracy and sensitivity to the risk dynamics using banks' financial performance at moderate frequency. Ramelli & Wagner (2020) identified firm characteristics that contribute to shock amplification, namely the composition of long- and short-term debt. They showed that firms with large cash holding are resilient to shocks, and, hence, an approach to capture such firms' characteristics is needed.

In this paper, we develop a joint default probability index to signal potential systemic risks in the highly concentrated Indonesian banking industry. Hansen (2014) warns that model misspecification can be a serious problem when devising systemic risk measures. Therefore, we estimate the banks' probability of default based on a multivariate distribution and the copula approach. The advantage of using the copula approach is that we are able to capture non-linear relationships between variables with complex data structures, where the dependency structure between two random variables that are asymmetrical (upper negative or upper positive). As argued by Patton (2006), the mainstream models for estimating indexes, such as systemic risk, assume the underlying relationships are linear. If this assumption is not satisfied, then estimated systemic risk measures are inaccurate or underestimate extreme events. Our approach circumvents this problem and is in line with Pourkhanali *et al.* (2016), Zhang (2014), and Brechmann *et al.* (2013), who use a copula approach to estimate dependence structure within the firms.

Our joint default probability index uses bank-level data extracted from Indonesian banks' financial statements. We estimate bank-level tail risks, using monthly data based on a sample of 104 banks from December 2003 to April 2020, in order to capture bank-specific characteristics. We focus on the Indonesian banking system because it has a relatively higher number of banks when compare to neighboring countries in the Southeast Asia region. Besides, the Indonesian banking system is dominated by domestic banks in terms of ownership type. The distribution of Indonesian banks is as follows: state-owned (4 banks), national private commercial (63 banks), regional development (27 banks), joint venture (12 banks), and foreign (9 banks). Singapore has more banks than Indonesia, but its banking system is dominated by foreign banks. In addition, the Indonesian banking system is highly dispersed, measuring size variability, in terms of total asset, capital, loan, or deposit between banks as shown by the concentration ratio of top five bank market shares in Indonesia is relatively high (i.e. 51.45% in December 2019). The unique characteristics of the Indonesian banking industry pose challenges in measuring systemic risk, as the high concentration ratio indicates the potential for the too-big-to-fail problem. Simultaneously, the significant number of banks with identical sizes indicates the potential for the too-many-too-fail problem. With that risk spectrum, a composite index reflecting potential joint default or potential systemic risk amongst banks will prove invaluable for macroprudential policy purposes.

The central bank of Indonesia (Bank Indonesia) considers the presence of shocks and vulnerabilities to estimate the incidence of systemic risk in the banking sector (Harun et al., 2015). The co-incidence of both would prevail when a systemic risk event occurs in the system. Bank Indonesia considers the size, interconnectedness, and complexity of the banking system to monitor potential systemic risk. Because of the complexity of systemic risk, it is obvious that Bank Indonesia needs an additional quantifiable indicator - an early warning indicator to monitor potential spillover risk to infer potential amplification in the dynamics of systemic risk at a considerably high frequency. At such a frequency, information is typically available in the stock market, where the default probability could be estimated using the Merton (1974) model. However, Merton's approach is very limited in the Indonesian case, since only fractions of banks are listed on the stock exchange. Moreover, Jacobs (2016) found that the stock market in Indonesia is highly mispriced, implying significant potential bias in market-based estimation. Because of these issues, we focus on the monthly financial reports of Indonesian banks to minimize bias in our index.

To bridge the gap between market mispricing and data disclosure, we use a non-linear approach to capture spillover risk between banks, indicating potential systemic risk. Using a copula approach, we find that non-conventional joint multivariate distribution, i.e. non-normal distribution, is a typical underlying process in the banks' fundamental information as reflected in their financial reports. Our findings indicate that individual banks' process is complex as it is captured from the distribution fitting process. Such a complex distribution and non-linearity in the banks' financial data underlie our individual default probability estimates, which are then aggregated as a composite index. Our approach can also explain the interconnected issues within the Indonesian banking system, as stated by Aini & Koesrindartoto (2020).

As robustness checks, the individual banks' default probability estimates are, to some extent, consistent with estimates obtained from the Merton model. The difference between these estimates is due to the information content of each model. Our bank-level default probability estimates can pick up early signals of rising default probability during the global financial crisis of 2007/2008. Our aggregated default probability (or composite) index for Indonesian banks captures a higher potential systemic risk before the COVID-19 global pandemic. This finding adds

to a growing literature showing an increase in uncertainty across global markets following the COVID-19 outbreak (see, for example, Devpura and Narayan, 2020; Haroon and Rizvi, 2020; Iyke, 2020a,b; Mishra et al., 2020; Narayan, 2020; Phan and Narayan, 2020; Prabheesh et al., 2020; Salisu and and Akanni, 2020; Vidya and Prabheesh, 2020). Furthermore, we also find that the state-owned banks that own more than 50% of the total asset in the Indonesian banking industry are highly volatile around the financial crisis between June 2007 to December 2008.

The rest of the paper is structured as follows. In section II, we discuss the pair copula construction used to develop the joint probability of default index for the Indonesian banking sector. Section III presents the empirical analysis and discussions. Section IV concludes.

II. DATA AND METHODOLOGY

We employ the pair copula construction (PCC) to develop our default risk index. This approach has the advantage that it eliminates spurious dependencies both during normal and extreme conditions. To identify the potential systemic risk from each bank, there are three steps for constructing the probability of default index using copula for the Indonesian banking system. First of all, we estimate the individual probability of default (PD) from each bank to snapshot the multivariate dependencies from financial variable in the balance sheet namely current assets, current liabilities, long-term assets, and long-term liabilities (see Valle *et al (2016)*). Secondly, we employ the rolling estimation to curb the dynamic PD for all bank samples. Finally, the PD index is the aggregate value from individual PD using a value weighted method.

A. Copula and Vine Copula

Copula approach is a dependency function of marginal form that allows combining two marginals into one function or joint distribution (Sklar, 1959). The copula is useful in understanding various fallacies related to correlation. In finance, copula can be used to determine the pricing of a financial asset and the nature of risky assets, particularly in isolating the dependence structure within a multivariate distribution. In our case, copula plays an important role in examining the relationship between firms, particularly when extreme events occur. Following Sklar (1959), consider a *d*-dimensional joint cumulative distribution function $F(x_1,...,x_d)$ and marginal cumulative distributions $F_1,...,F_d$. There exists a copula, *C*, such that

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d); \mathbf{\Theta})$$
(1)

for all $x_i \in [-\infty,\infty], i=1,...,d$. Θ denotes the set of parameters of the copula. If F_i is continuous for all i=1,...,d, then the *d*-dimensional copula is uniquely defined. The joint density function can be written as:

$$f(x_1, \dots, x_d) = c(F_1(x_1), \dots, F_d(x_d)) \cdot f_1(x_1) \cdots f_d(x_d)$$
(2)

where $c(F_1(x_1), \dots, F_d(x_d))$ denotes the *d*-variate copula density given its existence.

However, there are some issues related to the use of copula. The parameter estimation of joint multivariate distributions may not be accurate, when each variable does not have the same distribution. Besides, the use of copula is challenging in higher dimensions, where the standard multivariate copula suffers from inflexible structure and parameter restrictions.

To overcome this limitation and model complex dependency patterns from the broad range of bivariate copula, we employ the vine or pair copula construction approach to examine the dependence structure of the Indonesian financial sector. We follow Joe (1993), Bedford and Cooke (2001, 2002), Kurowicka and Cooke (2006), and Aas *et al.* (2009) to develop our empirical strategy. The vine copula is proposed by Joe (1996) and further developed by Bedford and Cooke (2001); it was developed by decomposing a cascade of the bivariate copula, known as PCC, to estimate multivariate copula from bivariate copula functions. Since PCC is chosen independently, it provides a significantly flexible framework for estimating the probability of defaults.

Furthermore, a vine is a graphical method to label constraints in distributions with a high dimension (see Bedford and Cooke, 2001; Cooke, 1997; Kurowicka and Cooke, 2006). To obtain the vine copula, first, we factorize the joint distribution $f(x_1, ..., x_d)$ of the random vector $X=X_1, ..., X_d$ as a product of conditional densities as follows

$$f(x_1, \dots, x_d) = f_d(x_d) \cdot f_{d-1|d}(x_{d-1}|x_d) \cdot \dots \cdot f_{1|2-d}(x_1|x_2, \dots, x_d).$$
(3)

Using Sklar's theorem, the joint distribution of the subvector $(X_{d'}X_{d-1})$ can be written in terms of a copula density as follows

$$f(x_{d-1}, x_d) = c_{d-1,d}(F_{d-1}(x_{d-1}), F_d(x_d)) \cdot f_{d-1}(x_{d-1}) \cdot f_d(x_d)$$
(4)

where $c_{d-1,d}$ denotes an arbitrary bivariate copula or pair copula density. For an element of X₁ of the vector **X**, we can get:

$$f_{X_{J}|\boldsymbol{\nu}}(X_{J}|\boldsymbol{\nu}) = c_{X_{J}|\boldsymbol{\nu}_{-\ell}}(F_{X_{J}|\boldsymbol{\nu}_{-\ell}}(X_{J}|\boldsymbol{\nu}_{-\ell}), F_{\boldsymbol{\nu}_{\ell}|\boldsymbol{\nu}_{\ell}}(\boldsymbol{\nu}_{\ell}|\boldsymbol{\nu}_{\ell})) \cdot f_{X_{J}|\boldsymbol{\nu}_{-\ell}}(X_{J}|\boldsymbol{\nu}_{-\ell})$$
(5)

where v denotes the conditioning vector, v_i denotes a generic component of v, v_j denotes the vector v without component $v_{i'} F_{X_j | v_{-\ell}}(\cdot | \cdot)$ denotes the conditional distribution of x_j given $v_{,i'}$ and $c_{X_j | v_{-\ell}}(\cdot, \cdot)$ denotes the conditional pair copula density.

The PCC is then constructed by decomposing the *d*-dimensional joint multivariate distribution function into a product of bivariate copulas and marginal distributions by recursively plugging Equation (5) in (3). Since the PCC is order dependent, the choice of variable order is very important. The choice will determine the PCC and the factorization of the joint multivariate distribution. For this reason, it is important to determine a suitable representation of a high distribution in the PCC.

Bedford and Cooke (2001, 2002) propose regular vines (R-vines) as a pictorial representation of PCC. Since a vine *V* on *n* variables can be described as a nested

set of connected trees $V=\{T_1,...,T_{n-1}\}$, where the edges of the tree *j* are the nodes, tree *j*+1,*j*=1,...,*n*-2, an R-vine can be defined as a special case in which all constraints are two-dimensional or conditional two-dimensional. In this case, a regular vine on *n* variables is a vine in which two edges are tree *j* joined by an edge in tree *j*+1 only if these edges share a common node *j*=1,...,*n*-2. An R-vine is called a canonical or *C*-vine if each tree *T_i* has a unique node of degree *n*-*i*, and therefore has the maximum degree. The *C*-vine is an R-vine that has a dependence center. A R-vine is called *D*-vine or drawable vine if all nodes in *T*₁ have degrees no higher than 2 (see Cooke *et al.*, 2011).

B. Data

We use balance sheet data from each bank consisting of current assets, long-term assets, current liabilities, and long-term liabilities, to estimate the probability of default. We define the account name of the balance sheet of a bank as follows: current asset (CA) is a bank asset that has a maturity of less than one year (liquid assets), long-term asset (LA) is a bank asset that has a maturity of more than one year, current liabilities (LL) is a bank liability that has a maturity of less than one year, and long-term liabilities (LL) is a bank liability which has a maturity of more than one year.

All CAs and LAs are in terms of their net values, respectively, meaning that the value of assets is deducted by loan loss provision and depreciation. All financial data of Indonesian banks are retrieved from Bank Indonesia's database. We use monthly data of all banks in Indonesia from the period September 2000 to April 2020 (equivalent of 236 months) from Bank Indonesia's database. Our sample consists of 104 banks, leading to 24,544 bank–month observations. To further analyze the characteristics of defaults, we separate our sample into four bank groups, based on the type of ownership used mainly in Indonesia, namely the state-owned banks (or *bank persero*), private banks (or *bank umum swasta nasional*), foreign banks (or *kantor cabang bank asing*), and regional development banks (or *bank pembangunan daerah*).

C. Individual Probability of Defaults

At this stage, we examine the probability of default for each bank based on balance sheet characteristics. The steps are: (1) we determine the marginal distributions, (2) we select the dependence structure (tree) and choose the appropriate copula families, (3) we conduct simulations to get equity value estimates, (4) we estimate the inverse function from pseudo observations to original observations, and (5) we estimate the probability of default, whereby the probability is taken from negative equity values. To estimate the dynamics of firm value, we use PCC as described by Valle *et al.* (2016). The basic model is a contingent claim model, where the underlying securities are equity and debt of a bank. The balance sheet data can be used to derive a proxy for the market value of a bank. The "latent" value of the bank is given by $A_T = G(E_T, B_T; T)$, where $G(\cdot)$ is the pay-off function, A_T, E_T, B_T denote asset, equity, and debt for period T, respectively. We intuitively expand Equation (5) into:

$$E_T = G_1(A_t, B_t; t) = P(t, T) \int_0^\infty \int_0^\infty G_1(A_T, B_T; T) g_1(A_T, B_T) dA_T dB_T$$
(6)

Using the Sklar's theorem, the realization of the data on the balance sheet is mapped into the copula function $c(\cdot)$:

$$E_{t} = P(t,T) \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} G_{2}(A_{C_{T}}, A_{L_{T}}, B_{C_{T}}, B_{L_{T}}; T) \cdot c(F_{A_{C}}, F_{A_{L}}, F_{B_{C}}, F_{B_{L}}) f_{A_{C}} f_{A_{L}} f_{B_{C}} f_{B_{L}} dA_{C_{T}} dA_{L_{T}} dB_{C_{T}} dB_{L_{T}}$$
(7)

where $c(\cdot)$ denotes four-dimensional copula density function, $F(\cdot)$ denotes marginal cumulative distribution function, and $f(\cdot)$ denotes marginal probability density function.

Using Monte-Carlo simulation, the values of a bank's equity can be estimated as follows:

$$\widetilde{E}_{t} = P(t,T) \frac{1}{N} \sum_{k=1}^{N} G_{2}(\widetilde{A}_{C_{Tk}}, \widetilde{A}_{L_{Tk}}, \widetilde{B}_{C_{Tk}}, \widetilde{B}_{L_{Tk}}; T)$$
(8)

An inverse function from a uniform distribution to a real distribution can be estimated as follows:

$$C(u_1, \dots, u_d) = F(F_1^{-1}(u_1), \dots, F_d^{-1}(u_d)).$$
(9)

After obtaining the estimates of the probability of defaults for each bank, we construct a time series probability of default across the observation period by using a rolling estimation with a 36-month window. Considering our relatively shorter sample period, we use a 36-month window is to get optimal values. This enables us to get the estimated probability of defaults dynamically.

D. Systemic Risk Identification

Correlated default can be defined as a condition where the default of one bank has a strong relationship or dependency with another bank. This relationship indicates that an increase in the probability of default in one bank will increase the default probability in related banks and other financial institutions (see Benoit *et al.*, 2017). However, from a macroprudential perspective, monitoring default correlations between banks in a financial system is cumbersome. In addition, the supervision of specific bank groups that contribute largely to default risk correlation is considered to be more effective because these bank groups have a higher potential systemic risk. Pourkhanali *et al.* (2016) propose an approach to identify potentially systemic risk in such banks. This approach is based on the concept of partial correlation and examines what banks contribute significantly to increasing the value of the default correlation. Therefore, we estimate correlated default across the banks and conduct systemic risk analysis by using partial correlations as follows:

$$\rho_{YX|Z_{1},Z_{2}...,Z_{n}} = \frac{\rho_{YX|Z_{1},Z_{2}...,Z_{n-1}} - \left(\rho_{YZ_{n}|Z_{1},Z_{2}...,Z_{n-1}}\right)\left(\rho_{XZ_{n}|Z_{1},Z_{2}...,Z_{n-1}}\right)}{\sqrt{\left(1 - \rho_{YZ_{n}|Z_{1},Z_{2}...,Z_{n-1}}^{2}\right)\left(1 - \rho_{XZ_{n}|Z_{1},Z_{2}...,Z_{n-1}}^{2}\right)}}$$
(10)

where ρ denotes partial correlation.

To illustrate this concept, Figure 1 describes the four banks that have interdependent relationships, whereby Bank 3 is the center of the dependencies. For example, the default correlation between Bank 1 and Bank 2 has a large value of 90%, and after the relationship is controlled (through partial correlation) by Bank 3, the dependency value becomes -10%. Therefore, based on this example, Bank 3 has the potential to be systemic because it causes a higher default correlation on Bank 1 and Bank 2.



In the last stage, we construct an index to identify potential systemic risk within the banking sector. Following Acharya *et al.* (2014) and Rosenberg and Schuermann (2006), the probability of default index for the banking sector is constructed as an aggregation of the probability of default from four bank groups, namely the state-owned banks (*ST*), private banks (*PR*), foreign banks (*FO*), and regional development banks (*RG*). The aggregate probability of default for each group is created using a value-weighted approach based on the standard deviation of total assets. The probability of defaults index is computed using the Bayesian approach, whereby the weight of each bank group is determined based on the standard deviation:

$$Index_{PD_t} = \sum_{i=1}^{N} w_{type,t} PD_{type,t}$$
(11)

where $w_{type,t} = \frac{I_{type,t}}{I_{ST,t} + I_{PR,t} + I_{FO,t} + I_{RG,t}}$ and $I_{type,t} = \frac{1}{\sigma_{PD_{type,t}}}$.

III. MAIN FINDINGS

This section comprises three subsections. We discuss several key statistical features of the balance sheet data, the probability of default index construction, and the empirical results of the individual probability of default in the first subsection, followed by the results for bank groups in the second subsection. The final subsection discusses the correlated default indexes and the identification of potential systemic risk in the Indonesian banking sector.

A. Statistical Features of the Balance Sheet Data and the Individual Probability of Default Before going on to the empirical analysis, we conducted some treatments for our sample by filtering the Inter-Office Accounts (*Rekening Antar Kantor*) to remove high dependencies on financial statement data decomposition. The descriptive statistics for the banks are presented in Tables 1 and 2.

Table 1. Descriptive Statistics for All Bank Samples

This table reports descriptive statistics of the Indonesian banking sector from 2000 until 2020 using monthly financial statement figures. The total samples used are 104 banks. All values are in a million Indonesian rupiah. *Source*: authors' calculations.

	(a) Descriptive sta	itistics for all ban	ks based on balance she	et item
	Current assets	Long-term assets	Current-liabilities	Long-term liabilities
Mean	11,498,955	21,389,916	26,037,931	1,953,758
Median	2,033,741	3,506,227	4,681,905	175,199
St. Deviation	32,135,823	66,935,010	77,077,536	6,510,745
Min	0	0	0	0
Max	340,768,408	810,491,856	896,479,220	119,940,733
	(b) The average	value for each ba	lance sheet item for each	n year
Year	Current assets	Long-term assets	Current-liabilities	Long-term liabilities
2000	6,165,953	2,328,524	6,995,104	1,299,912
2001	6,136,246	2,878,112	7,387,911	1,293,215
2002	6,215,978	3,194,008	7,919,663	727,421
2003	6,029,358	3,894,621	8,354,438	625,652
2004	5,769,392	4,772,304	8,806,358	607,390
2005	5,926,141	6,201,193	10,215,780	636,768
2006	6,957,080	7,028,101	11,767,917	694,638
2007	7,899,266	8,365,240	13,651,734	801,082
2008	7,600,196	11,199,359	15,767,532	947,626
2009	8,876,456	13,190,539	18,462,187	1,071,764
2010	10,535,497	16,750,714	22,413,479	1,409,214
2011	11,174,330	19,383,767	24,691,239	1,690,110
2012	12,727,670	23,968,773	29,614,861	1,969,228
2013	13,356,249	29,209,657	33,960,504	2,378,113
2014	14,941,665	33,984,894	38,420,779	2,859,175
2015	17,920,647	37,636,933	43,359,191	3,328,236

	-		-	
	(b) The average	value for each ba	lance sheet item for each	year
Voar	Current	Long-term	Current-liabilities	Long-term
Ical	assets	assets	Current-Habilities	liabilities
2016	18,541,351	41,178,330	45,699,113	3,649,598
2017	20,981,692	44,918,033	50,191,143	4,258,950
2018	20,214,117	48,083,602	51,933,626	4,587,021
2019	16,493,480	48,329,606	50,331,002	3,211,808
2020	17,991,898	50,224,956	53,193,827	3,896,227

Table 1. Descriptive Statistics for All Bank Samples (Continued)

Table 2.Descriptive statistics based on bank groups

This table reports descriptive statistics of the Indonesian banking sector from 2000 until 2020 using monthly financial statement figures based on the type of ownership. The total samples used are 104 banks. All values are in million Indonesian rupiahs. *Source*: authors' calculations.

		State-Owned Bank		
	Current assets	Long-term assets	Current- liabilities	Long-term liabilities
Mean	115,869,447	218,407,424	266,811,941	24,225,108
Median	98,097,391	124,737,840	204,871,247	17,269,825
St. Deviation	83,898,151	212,117,628	221,411,087	20,116,468
Min	1,310,586	7,831,607	18,375,375	3,154,245
Max	340,768,408	810,491,856	896,479,220	119,940,733
		Private Bank		
	Current assets	Long-term assets	Current- liabilities	Long-term liabilities
Mean	7,943,522	15,858,552	19,060,289	1,251,720
Median	1,218,535	2,676,686	3,237,573	91,795
St. Deviation	21,782,212	41,814,214	50,742,152	3,169,601
Min	0	0	0	0
Max	313,358,318	593,247,013	696,105,336	39,147,998
		Foreign Bank		
	Current assets	Long-term assets	Current- liabilities	Long-term liabilities
Mean	11,824,270	14,655,391	15,739,859	1,100,996
Median	8,251,587	6,745,271	10,069,473	711,716
St. Deviation	11,433,187	19,599,672	15,394,139	1,291,322
Min	13,259	1,975	164,194	4,274
Max	55,746,436	112,612,428	74,544,523	8,832,424
	Reg	gional Development	Bank	
	Current assets	Long-term assets	Current- liabilities	Long-term liabilities
Mean	4,489,796	7,105,103	9,696,873	560,821
Median	2,673,879	3,392,018	5,392,376	247,963
St. Deviation	5,428,413	10,487,427	12,877,573	1,034,794
Min	10,334	13,367	17,614	313
Max	39,721,124	90,406,937	94,969,749	14,932,530

Figure 2 shows the distribution of the balance sheet accounts based on the aggregated data, using five distributional assumptions, namely lognormal, gamma, exponential, normal, and Weibull. We further fit the distributions for each balance sheet accounts (i.e. current assets, current liabilities, long-term assets, and long-term liabilities) using Kolmogorov-Smirnov and Andersen-Darling tests.









Source: authors' calculations.

B. Probability of Default for Individual Banks

In this section, we use data from the sample of two banks namely Bank Rakyat Indonesia (BBRI) and Bank Mandiri (BMRI) as an illustration that individual banks have different characteristics. Table 5 provides the results of the PCC estimation using the R-vine model. The table shows that in each tree, the optimal copula family values and parameters (parameters 1 and 2) are obtained for BBRI and BMRI, respectively. We can infer that although both banks have the same final tree structure, as shown in Tree 3, the distribution and dependence structure is different.

399

Table 3. The Estimation Results of Pair Copula Construction (PCC) for BBRI and BMRI Using Balance Sheet Accounts

This table reports the estimation results of pair copula construction (PCC) using balance sheet data from BBRI and BMRI between 2004 and 2020. For the tree column, 1 denotes current assets, 2 denotes long-term assets, 3 denotes current liabilities, and 4 denotes long-term liabilities. Source: authors' calculations.

			(a) BBRI			
Tree	Edge	Family	Copula	Parameter	Parameter 2	τ
1	4,3	10	BB8 (Frank-Joe)	5.948	0.965	0.701
1	3,1	5	Frank	32.155	0.000	0.882
1	3,2	5	Frank	100.000	0.000	0.961
2	4,1;3	1	Gaussian	0.261	0.000	0.168
2	1,2;3	36	Rotated Joe copula (270 degrees)	-1.406	0.000	-0.186
3	4,2;1,3	214	Tawn2 (180 degrees)	1.675	0.396	0.216
			(b) BMRI			
Tree	Edge	Family	Copula	Parameter	Parameter 2	·
1	4,1	5	Frank	12.589	0.000	0.724
1	1,3	10	BB8 (Frank-Joe)	5.272	0.896	0.623
1	3,2	6	Joe	10.121	0.000	0.824
2	4,3;1	104	Tawn type 1	2.526	0.318	0.248
2	1,2;3	40	Rotated BB8 copula (270 degrees)	-3.638	-0.872	-0.478
3	4,2;1,3	1	Gaussian	0.407	0.000	0.267

The selected vine copula model for BBRI is the C-vine model with a dependency center, while the dependence structure of BMRI is captured by the D-vine copula. This difference in the BBRI and BMRI results is due to the difference in their business models. Whereas BBRI focuses on retail banking, BMRI focuses on commercial banking. BBRI relies on its current liabilities, which become central to its current assets, long-term assets, and long-term liabilities (see the structure of Tree 1 for BBRI and Figure 3). For BMRI, current assets and currents liabilities have a strong dependency; at the same time, current assets are related to longterm liabilities and current liabilities related to long term assets (see the structure of Tree 1 for BMRI and Figure 4). The dependency value for the bivariate copula is strongest for BBRI in the first tree compared to the second and third trees. In the first tree, the highest dependency value is for the long-term asset, and the current liabilities with the copula family is Frank, and the value of Kendall's tau is 0.96.

Figure 3. Tree Structure for BBRI. Source: Authors' Calculations





After obtaining the results using PCC, we simulate, for each variable (i.e. *CA*, *LA*, *CL*, and *LL*), the process 10,000 times using Monte Carlo simulation. The simulated results are then used to determine the value of the probability of default for each bank. The probability of default value is obtained from the equity density when the value is less than 0. We find that the probability of default values for BBRI and BMRI are, respectively, 16.58% and 16.49%. We use the same procedure to obtain the probability of default for the rest of the banks in our sample.

C. Probability of Default based on Bank Groups

After conducting all necessary steps to obtain the probability of default for all banks, as described in the previous section using two banks as examples, we compute the descriptive statistics for all banks based on bank types. Table 4 provides the descriptive statistics on the probability of default for each bank type or group. We can see that regional development banks (BPD), on average, have a higher probability of default across the sample periods, while the private banks have a lower probability of default relative to other banks.

Figure 5 depicts a comparison of the probability of default for each bank group. It is interesting to see that before the global financial crisis of 2007-2008, stateowned banks have a higher value of the probability of default compare to other banks. After the crisis, the probability of default for regional development banks shows an increasing trend until the end of the sample period. Furthermore, the gap in the probability of default between regional development banks and banks is increasing over time. The state-owned banks indeed have more stable conditions, particularly after 2013, but their probability of default drastically increased in early 2020, possibly due to the COVID-19 pandemic, which has disrupted global economic activity (see Iyke, 2020a,b; Phan and Narayan, 2020).

developme	ent ba:	nks. The	estimation n	nethod is	s pair copul	la cons	truction (PCC) using	t balance	sheet da	ita fro	m 104 Inc	donesian b	anks betv	ween 20	04 and	1 2020. Sou	<i>irce</i> : autho	urs' calcu	lations.
		Sti	ate-Owne	d Bank			P	rivate Ba	nk			Fo	reign B	ank		Reg	ional -]	Develop	ment]	Bank
	Z	Mean	St. Dev	Min	Max	Ζ	Mean	St. Dev	Min	Max	Z	Mean	St. Dev	Min	Max	Z	Mean	St. Dev	Min	Max
2004	4	0.416	0.054	0.288	0.494	58	0.279	0.102	0.040	0.527	18	0.219	0.110	0.008	0.506	24	0.311	0.061	0.172	0.475
2005	4	0.379	0.064	0.239	0.498	58	0.277	0.106	0.012	0.554	18	0.218	0.115	0.014	0.490	24	0.315	0.063	0.206	0.475
2006	4	0.341	090.0	0.236	0.487	58	0.274	0.099	0.075	0.546	18	0.210	0.111	0.029	0.447	24	0.274	0.056	0.158	0.457
2007	4	0.299	0.049	0.210	0.381	58	0.281	0.102	0.054	0.557	18	0.213	0.110	0.011	0.458	24	0.241	0.045	0.137	0.368
2008	4	0.262	0.055	0.176	0.362	58	0.279	0.097	0.018	0.551	18	0.215	0.099	0.058	0.448	24	0.286	0.064	0.164	0.470
2009	4	0.225	0.054	0.146	0.320	58	0.274	0.098	0.008	0.474	18	0.189	0.077	0.026	0.393	24	0.320	0.068	0.167	0.525
2010	4	0.218	0.048	0.151	0.294	58	0.262	0.102	0.004	0.476	18	0.185	0.080	0.025	0.380	24	0.339	0.071	0.150	0.472
2011	4	0.201	0.039	0.133	0.295	58	0.250	0.096	0.021	0.504	18	0.201	0.091	0.026	0.383	24	0.297	0.079	0.150	0.497
2012	4	0.205	0.033	0.138	0.248	58	0.224	0.087	0.020	0.504	18	0.205	0.097	0.036	0.428	24	0.259	0.065	0.151	0.517
2013	4	0.200	0.052	0.115	0.293	58	0.219	0.089	0.011	0.563	18	0.178	0.078	0.031	0.422	24	0.263	0.052	0.161	0.399
2014	4	0.187	0.056	0.108	0.302	58	0.232	0.098	0.003	0.495	18	0.171	0.074	0.015	0.333	24	0.298	0.071	0.164	0.476
2015	4	0.191	0.044	0.106	0.278	58	0.232	0.091	0.034	0.502	18	0.182	0.072	0.045	0.381	24	0.300	0.076	0.138	0.510
2016	4	0.206	0.034	0.149	0.290	58	0.247	0.093	0.006	0.530	18	0.202	0.088	0.057	0.425	24	0.323	0.055	0.168	0.480
2017	4	0.204	0.042	0.134	0.287	58	0.266	0.085	0.008	0.495	18	0.211	0.091	0.026	0.414	24	0.355	0.065	0.206	0.536
2018	4	0.191	0.037	0.137	0.271	58	0.276	0.080	0.003	0.444	18	0.202	0.094	0.025	0.414	24	0.355	0.068	0.158	0.532
2019	4	0.222	0.063	0.148	0.562	58	0.292	0.092	0.028	0.508	18	0.220	0.115	0.016	0.499	24	0.330	0.072	0.131	0.502
2020	4	0.286	0.044	0.239	0.354	58	0.289	0.092	0.026	0.529	18	0.231	0.107	0.065	0.460	24	0.322	0.069	0.188	0.458

 Table 4.

 Descriptive statistics of the probability of defaults based on the type of ownership



Figure 6 shows the results of the probability of default for all banks and 30 large banks in Indonesia between December 2003 and December 2018 based on rolling estimation. The probability of default for almost all large banks increased to nearcrisis conditions from 2007 to the end of 2009. To better visualise our results, we used the Fed's quantitative easing policies of 2009 to 2015 as the timeframe. During this period, potential systemic risk is relatively lower compared to other periods. After the crisis, large banks experienced a decline in the probability of default, starting in early 2011 until the end of 2015; the probability of default gradually increased after this period until the end of the sample.

Figure 6. Probability of Default Estimates for All Banks and 30 Big Banks.

Note: The probability of default index is calculated using the value-weighted of probability of defaults for each bank. The dashed line indicates the time when the global financial crisis started. QE denotes FED's quantitative easing. Source: authors' calculations.



We also compare the probability of default estimates derived from the copula approach and to those derived from Merton's model. Figure 7 compares these probability of default estimates. Since the Merton model is based on the market value of respective bank stocks in the capital market, the values are significantly different compare to the results using the copula approach. We argue that the results using the copula approach are robust and more accurate, since the empirical estimations are based on the fundamental value of each bank.

Figure 7. Probability of Default Estimates using the Constructed Index and Merton Model



Source: authors' calculations.

D. Correlated Probability of Default

We estimate the joint probability of default indexes and their contribution to systemic risk with partial correlation and using the vine copula. Pourkhanali *et al.* (2016) argue that partial correlation is a joint distribution of two random variables that can capture the dependency structure without having to assume the two variables are independent of each other, unlike the canonical correlation, in general. The composite index of probability of default serves as an input to estimating the systemic risk contribution of banks via the partial correlation approach (see Equation (10)).

Figure 8 shows the dependence structure between different types of banks or bank groups. We can see the dependence structure within the groups. Stateowned banks and private banks are relatively more important than joint-venture and foreign banks, since they connect other banks within the banking system. The details of R-vine properties for different bank groups are given in Table 5.

Figure 8. R-Vine Tree 1 Based on Bank Groups

Note: 1 denotes state-owned banks, 2 denotes private banks, 3 denotes foreign banks, and 4 denotes regional development banks. Source: authors' calculations.



Table 5. Summary of PCC R-vine Construction Properties for Bank Groups

This table reports summary statistics of the PCC estimation results based on four types of ownership. 1 denotes stateowned banks, 2 denotes private banks, 3 denotes foreign banks, and 4 denotes regional development banks. *Source*: authors' calculations.

Tree	Edge	Family	Copula	Parameter	τ
1	4,2	1	Gaussian	0.427	0.281
1	2,1	1	Gaussian	0.496	0.330
1	1,3	1	Gaussian	-0.556	-0.375
2	4,1;2	1	Gaussian	-0.346	-0.225
2	2,3;1	1	Gaussian	0.168	0.108
3	4,3;2,1	1	Gaussian	0.319	0.207

Since we want to observe all correlated probability of default indexes between the bank groups, the PCC may not be adequate. Hence, we estimate the correlation and partial correlation using Kendall's τ , as shown in Table 6. Panel c of Table 6 shows that the correlation dependence between state-owned banks, private banks, and regional banks is relatively high, -56% (see 4,1;2). After conditioning, the correlation has decreased to 17%. In other words, private banks in Indonesia can balance the potential risk within the banking sector.

Table 6. Correlated Defaults Between Bank Groups

This table reports correlated defaults based on four types of banks: 1 denotes state-owned banks, 2 denotes private banks, 3 denotes foreign banks, and 4 denotes regional development banks. The correlated defaults are derived from partial correlation and correlation. *Source*: authors' calculations.

		(a) Correla	ation		
	State-owned	Private	Foreign	Regional	
State-owned	1.000				
Private	0.496	1.000			
Foreign	-0.555	-0.154	1.000		
Regional	-0.061	0.427	0.329	1.000	

COIL	cluten Dellullo	Detween		ips (comm	ucu)	
	(b)	Partial con	rrelation			
	State-owned	Private	Foreign	Regional		
State-owned	1.000		·			
Private	0.319	1.000				
Foreign	0.168	-0.346	1.000			
Regional	-0.555	0.495	0.427	1.000		
	(c)	Correlated	defaults			
	4,2	2,1	1,3	4,1 2	2,31	4,3 2,1
Correlation	-0.061	0.427	0.329	-0.556	-0.154	0.496
Partial correlation	-0.555	0.495	0.427	0.168	-0.346	0.319

Table 6.
Correlated Defaults Between Bank Groups (Continued)

E. Robustness Checks

To check the robustness of our findings, we compare our estimates of the probability of default to those derived using the Merton approach. We also check the correlation between each probability of default estimate and macroeconomic indicators. The correlation between copula estimates and credit growth is lower compare to the correlation between the Merton model estimates to credit growth. However, both have a similar correlation with the non-performing loan ratio (see Table 7, Panel a). This means that the copula provides the same credit risk information regarding credit performance.

We regress the probability of default estimates from the copula and Merton models on a macroeconomic indicator of business cycles, as described in Table 7 (Panel b). For this purpose, we use the credit-to-GDP gap ratio, which is an indicator commonly used by the Bank of International Settlement and central banks to measure countercyclical capital buffer (see Drehmann & Tsatsaronis, 2014). Panel b shows that our estimates based on the copula approach are consistent with those based on the Merton model. The correlation between the estimates from the copula approach and the Merton model, which is 43%, shows that the components are similar between these estimates. However, the adjusted *R*-squared shows that the probability of default estimates based on the copula approach are able to better capture the information content of banks (i.e. bank characteristics) than those based on the Merton model at 41% and 35% respectively. This shows that the copula approach is more precise in explaining the macroeconomic fundamentals.

Table 7. Robustness check results

This table reports the correlation and regression estimates between two probability of default estimates (based on copula and Merton model) and macroprudential indicators. PD Copula_42 denotes the probability of default estimated using copula from 42 bank's sample listed in Indonesia stock market, PD Copula_Full denotes the probability of default estimated using Merton model from 42 bank's listed in Indonesia, PD Merton denotes the probability of default estimated using Merton model from 42 bank's listed in Indonesia stock market, Credit Growth denotes the probability of default estimated using Merton model from 42 bank's listed in Indonesia stock market, Credit Growth denotes the probability of default estimated using industry to the economy, NPL denotes non-performing loan, and CGDP Gap denotes credit-to-GDP gap ratio. Finally, *** denotes p-values ≤ 0.001 . The *t*-values are in parentheses.

(a) Correla	tion between t	wo probability o indica	f default esti ators	mates and 1	nacroprud	ential
	PoD Copula_42	PoD Copula_ Full	PoD Merton	Credit Growth	NPL	CGDP Gap
PD Copula_42	1					
PD Copula_Full	0.8037	1				
PoD Merton	0.1558	0.4341	1			
Credit Growth	-0.0863	0.0555	0.5555	1		
NPL	0.485	0.5856	0.5885	0.1769	1	
CGDP Gap	0.3321	0.6467	0.6034	0.5196	0.2242	1
(b) Regression	results betwee	en two probabilit	y of default o	estimates ar	nd the cred	it-to-GDP

			ga	zap ratio
	CGDP		CGDP	
	Gap		Gap	
Intercept	-0.108	***	-0.0215	***
	(-6.2801)		(-3.989)	
PD Copula_Full	0.4822	***		
-	-6.7837			
PD Merton			0.2784	***
			-6.0538	
R-Squared	0.4183		0.3641	
Adj R-Squared	0.4092		0.3542	
F-Stat	46.0182		36.6482	

IV. CONCLUDING REMARKS

We study the probability of default of all Indonesian banks between September 2000 to April 2020, using monthly financial statements obtained from Bank Indonesia. Due to the mispricing issue in the capital market and data disclosure, a copula approach, namely the pair copula construction (PCC), is employed to derive the probability of default estimates, and then to construct a rolling based estimation of dynamic probability of default indexes. We find a joint multivariate dependency within the Indonesian banking system, which means normal distribution can underestimate the probability of default for state-owned banks, private banks, and foreign banks began to converge after the 2007-2008 global financial crisis until 2013. Since 2013, the probability of default for state-owned banks has been on the rise. Similarly, the probability of default for state-owned banks increase significantly near the COVID pandemic outbreak. The probability of default for regional

development banks is the highest during the estimation period. These results show that our copula-based joint probability of default index is a good indicator of systemic risk within the banking industry.

We examine the PD copula and the credit-to-GDP gap ratio to see the relationship of PD to the economy. The correlation results indicate that the PD copula estimates are strongly related to the direction of the economy. Furthermore, the regression results show that PD Copula is significant to explain the credit-to-GDP gap ratio. As a robustness check, we compare our estimates to those derived from the Merton model, which uses information on stock markets. We also address interconnected issues by mapping the relationship between the four bank groups. We find that our approach has better power in explaining the role of systemic risk in an economy dominated by the banking industry. Our joint probability of default index can be used as an informative signal to enhance Bank Indonesia's macroprudential supervisory framework.

REFERENCES

- Aas, K., Czado, C., Frigessi, A., & Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and Economics*, 44, 182–198.
- Acharya, V., Engle, R., & Pierret, D. (2014). Testing Macroprudential Stress Tests: The Risk of Regulatory Risk Weights. *Journal of Monetary Economics*, 65, 36–53.
- Aini, M., & Koesrindartoto, D. P. (2020). The Determinants of Systemic Risk: Evidence from Indonesian Commercial Banks. *Buletin Ekonomi Moneter Dan Perbankan*, 23, 101–120.
- Basel Committee on Banking Supervision. (2018). Global systemically Important Banks: Revised Assessment Methodology and the Higher Loss Absorbency Requirement. https://www.bis.org/bcbs/publ/d445.htm
- Bedford, T., & Cooke, R. M. (2001). Probability Density Decomposition for Conditionally Dependent Random Variables Modeled by Vines. Annals of Mathematics and Artificial Intelligence, 32, 245–268.
- Bedford, T., & Cooke, R. M. (2002). Vines: A new graphical model for dependent random variables. *The Annals of Statistics*, 30, 1031–1068.
- Benoit, S., Colliard, J.-E., Hurlin, C., & Pérignon, C. (2017). Where the risks lie: A survey on systemic risk. *Review of Finance*, 21, 109–152.
- Bisias, D., Flood, M., Lo, A. W., & Valavanis, S. (2012). A survey of systemic risk analytics. *Annual Review of Financial Economics*, 4, 255–296.
- Black, F. (1986). Noise. The Journal of Finance, 41, 528-543.
- Brechmann, E. C., Hendrich, K., & Czado, C. (2013). Conditional copula simulation for systemic risk stress testing. *Insurance: Mathematics and Economics*, 53, 722– 732.
- Cooke, R. M. (1997). Markov and Entropy Properties of Tree- and Vine-Dependent Variables. *Proceedings of the Section on Bayesian Statistical Science* (Vol. 27). American Statistical Association.
- Cooke, R. M., Joe, H., & Aas, K. (2011). Vines Arise. In D. Kurowicka (Ed.), Dependence Modeling: Vine Copula Handbook (pp. 38–71). World Scientific Publishing Co Pte Ltd.

- Devpura, N., & Narayan, P.K., (2020) Hourly Oil Price Volatility: The role of COVID-19. *Energy Research Letters*, 1(2), 13683. https://doi. org/10.46557/001c.13683.
- Drehmann, M., & Tsatsaronis, K. (2014). The Credit-To-GDP Gap and Countercyclical Capital Buffers: Questions and Answers. *BIS Quarterly Review*. March, 55-73.
- Greenwood, R., Landier, A., & Thesmar, D. (2015). Vulnerable banks. *Journal of Financial Economics*, 115, 471–485.
- Hansen, L. P. (2014). Challenges in Identifying and Measuring Systemic Risk. In M. K. Brunnermeier & A. Krishnamurthy (Eds.), *Risk Topography: Systemic Risk* and Macro Modeling. The University of Chicago Press.
- Haroon, O., and Rizvi, S.A.R. (2020). Flatten the Curve and Stock Market Liquidity—An Inquiry Into Emerging Economies. *Emerging Markets Finance and Trade*, 56, 2151-2161.
- Harun, C. A., Rachmanira, S., & Nattan, R. R. (2015). Kerangka Pengukuran Risiko Sistemik (Bank Indonesia Occasional Paper, OP/4/2015). Bank Indonesia. https://www.bi.go.id/id/publikasi/occasional-paper/Documents/Kerangka-Pengukuran-Risiko-Sistemik.pdf
- Henker, T., & Husodo, Z. A. (2010). Noise and efficient variance in the Indonesia Stock Exchange. *Pacific-Basin Finance Journal*, 18, 199–216.
- Iyke, B. (2020a). COVID-19: The Reaction of USOil and Gas Producers to the Pandemic. *Energy Research Letters*, 1(2), 13912. https://doi.org/10.46557/001c.13912.
- Iyke, B.N. (2020b). The Disease Outbreak Channel Of Exchange Rate Return Predictability: Evidence from COVID-19. *Emerging Markets Finance and Trade*, 56, 2277-2297.
- Jacobs, H. (2016). Market Maturity and Mispricing. *Journal of Financial Economics*, 122, 270–287.
- Joe, H. (1993). Parametric Families of Multivariate Distributions with Given Margins. *Journal of Multivariate Analysis*, 46, 262–282.
- Joe, H. (1996). Families of M-Variate Distributions with Given Margins and M(M-1)/2 Bivariate Dependence Parameters. *Institute of Mathematical Statistics Lecture Notes - Monograph Series*, 120–141.
- Juhro, S. M., & Iyke, B. N. (2019). Monetary Policy and Financial Conditions in Indonesia. *Buletin Ekonomi Moneter dan Perbankan*, 21, 283-302.
- Kurowicka, D., & Cooke, R. M. (2006). Uncertainty Analysis with High Dimensional Dependence Modelling. John Wiley & Sons, Ltd.
- Merton, R. C. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance*, 29, 449–470.
- Mishra, A.K., Rath, B.N., & Dash, A.K., (2020) Does the Indian financial market nosedive because of the COVID-19 outbreak, in comparison to after demonetisation and the GST? *Emerging Markets Finance and Trade*, 56, 2162-2180.
- Narayan, P., K. (2020). Oil price news and COVID-19—Is there any connection?. *Energy Research Letter*, 1, 1-5, 13176. https://doi.org/10.46557/001c.13176
- Patton, A. J. (2006). Modelling Asymmetric Exchange Rate Dependence. International Economic Review, 47, 527–556.

- Phan, D.H.B., & Narayan, P.K., (2020) Country Responses and the Reaction of the Stock Market to COVID-19—a Preliminary Exposition, *Emerging Markets Finance and Trade*, 56, 2138-2150.
- Pourkhanali, A., Kim, J.-M., Tafakori, L., & Fard, F. A. (2016). Measuring Systemic Risk Using Vine-Copula. *Economic Modelling*, 53, 63–74.
- Prabheesh, K.P., Padhan, R., & Garg, B., (2020) COVID-19 and the oil price—stock market nexus: Evidence from net oil-importing countries, *Energy Research Letters*, 1, 13745. https://doi.org/10.46557/001c.13745
- Ramelli, S., & Wagner, A. F. (2020). Feverish Stock Price Reactions to the Novel Coronavirus. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.3550274
- Rosenberg, J. V., & Schuermann, T. (2006). A General Approach to Integrated Risk Management with Skewed, Fat-Tailed Risks. *Journal of Financial Economics*, 79, 569–614.
- Salisu, A., A & Akanni, L., O. (2020). Constructing a Global Fear Index for the COVID-19 Pandemic. *Emerging Markets Finance and Trade*, 56, 2310-2331.
- Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris*, 8, 229–231.
- Valle, D. L., De Giuli, M. E., Tarantola, C., & Manelli, C. (2016). Default probability estimation via pair copula constructions. *European Journal of Operational Research*, 249, 298–311.
- Vidya, C. T., and Prabheesh, K.P. (2020). Implications of COVID-19 Pandemic On The Global Trade Networks. *Emerging Markets Finance and Trade*, 56, 2408-2421.
- Zhang, D. (2014). Vine Copulas And Applications To The European Union Sovereign Debt Analysis. *International Review of Financial Analysis*, 36, 46–56.

This page is intentionally left blank